Student Number: ______________________ Class Teacher: ______________________

St George Girls High School

Trial Higher School Certificate Examination

2017

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 15, show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Pages 2 – 4

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Answer on the multiple choice answer sheet provided at the back of this paper

Section II Pages 5 – 10

90 marks

- Attempt Questions 11 – 15
- Allow about 2 hours and 45 minutes for this section
- Begin each question in a new writing booklet

Section I	**/10**
Section II	
Question 11	/18
Question 12	/18
Question 13	/18
Question 14	/18
Question 15	/18
Total	**/100**

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1-10

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1. The graph of $y = x(x^2 - 1)$ intersects with the x axis at:

   (A) 1 point
   (B) 2 points
   (C) 3 points
   (D) 4 points

2. Which of the following quadratic expressions is positive definite?

   (A) $x^2 + 7x + 1$
   (B) $x^2 + 7x - 1$
   (C) $x^2 + 7x + 15$
   (D) $x^2 + 7x - 15$

3. What is the <u>range</u> of the function $f(x) = \sqrt{4 - x^2}$

   (A) $0 < y < 2$
   (B) $0 \leq y \leq 2$
   (C) $-2 < y < 2$
   (D) $-2 \leq y \leq 2$

4. The focus of the parabola $x^2 = 8(y - 1)$ is at:

   (A) $(0, 1)$
   (B) $(0, 3)$
   (C) $(0, -1)$
   (D) $(0, 8)$

Section I (cont'd)

5. What is the period of $y = \tan 6x$.

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) 6π

(D) 12π

6. What is the value of $\int_{-4}^{3} |x+2| \; dx$

(A) $\frac{21}{2}$

(B) $\frac{53}{2}$

(C) $\frac{3}{2}$

(D) $\frac{29}{2}$

7. If $y = xe^{2x}$ then $\frac{dy}{dx} =$

(A) $x\, e^{2x}$

(B) $2x\, e^{2x}$

(C) $(1+2x)\, e^{2x}$

(D) $(1+x)\, e^{2x}$

8. $|2x+4| = -x+4$ when solved has:

(A) no solution

(B) 1 solution

(C) 2 solutions

(D) 3 solutions

Section I (cont'd)

9. If $f(x) = \frac{3x^4 - x}{x^2}$ then $f'(1) =$

(A) 5

(B) 7

(C) 0

(D) 2

10. $\sum_{r=1}^{5} (-1)^r \, 2^r =$

(A) 6

(B) -62

(C) 22

(D) -22

End of Section I

Section II

90 marks
Attempt Questions 11 – 15
Allow about 2 hours and 45 minutes for this section.

Start each question in a new writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (18 marks) Start a New Writing Booklet. **Marks**

a) Simplify $\sqrt{75} - \frac{1}{2}\sqrt{48}$. 1

b) Find to 2 decimal places $\sec 40°15'$. 1

c) Draw a neat sketch of $y = 3\cos 2x$ for $0 \le x \le 2\pi$,
showing clearly all relevant features. 2

d) A point $P(x, y)$ moves so that its distance from the x-axis is always
twice its distance from the y-axis. Describe this locus geometrically. 1

e) Find the radius of the circle $x^2 + 4x + y^2 - 6y - 12 = 0$. 2

f) Write in simplest form $\frac{x+3}{x^{-1}+3^{-1}}$. 1

g) Differentiate with respect to x

(i) $\log_e\sqrt{3x^2 - 2}$ (ii) $\frac{x+3}{2x-5}$ 4

h) Prove $\frac{1}{\sin\theta\cos\theta} - \tan\theta = \cot\theta$. 2

i) (i) Find the stationary points of the function $y = 2x^3 - 12x^2 + 18x - 3$
and determine their nature. 3

(ii) In the domain $\{x: -5 \le x \le 5\}$ what is the greatest value of
$2x^3 - 12x^2 + 18x - 3$? 1

Question 12 (18 marks) Start a New Writing Booklet. **Marks**

a) State the domain of x if $x = 3^y$. 1

b) Find the area between the curve $y = e^x - 2$ and the x-axis from $x = 0$ and $x = 3$. 3

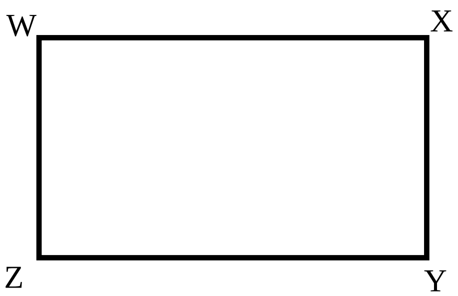
c) For the arithmetic sequence $400, 350, 300, \ldots$ find:

(i) An expression for T_n . 1

(ii) Which is the first negative term of the sequence? 2

(iii) The sum of the first 20 terms. 1

d) A particle moves along the x axis with acceleration $(t - 2)$ m/s^2 . Initially it is 1 m to the right of the origin, with velocity 3 m/s. What is the position of the particle after 6 seconds? 3

e) 

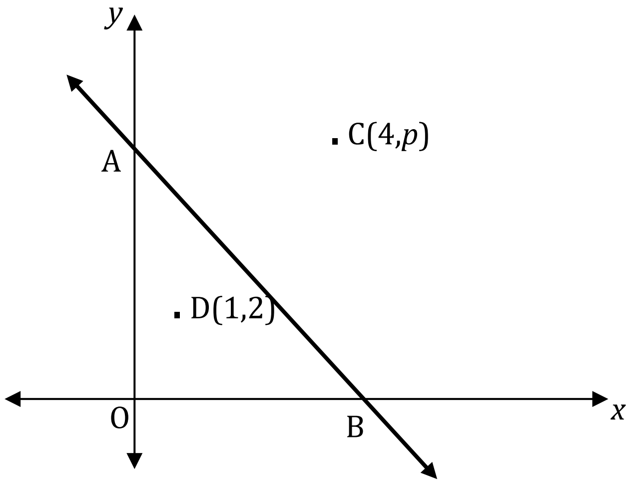
WXYZ is a rectangle. Prove XZ = WY. 3

f) Solve $x^2 > 3x$. 2

g) For what values of x is $y = x^3 - 3x + 5$ an increasing function? 2

Question 13 (18 marks) Start a New Writing Booklet. **Marks**

a)



The line $2x + 3y = 12$ cuts the x-axis at B and the y-axis at A.

(i) Calculate the length of AB as a simplified surd. 1

(ii) If AC is perpendicular to AB, find the value of p if C is the point $(4,p)$ and D $(1,2)$. 2

(iii) Calculate the perpendicular distance from D to AB. 2

(iv) Hence, or otherwise, find the area of ΔABD. 1

(v) Draw this diagram in your answer booklet and shade the region $2x + 3y < 12$. 1

b) If $0 \leq \theta \leq 2\pi$ solve $\sin 2\theta = -\dfrac{\sqrt{3}}{2}$. 2

c) If α and β are the roots of $3x^2 - 4x - 1 = 0$ find the values of:

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

(iii) $\alpha^2 + \beta^2$ 1

d) Use Simpson's Rule with 5 function values to approximate $\displaystyle\int_1^9 \log_e x \; dx$, giving your answer to 2 significant figures. 3

e) A particle moving in a straight line at time t (in seconds) has displacement x (in cm) given by $x = 6t - t^3$. When is the particle at rest and what is the acceleration at that time? 3

Question 14 (18 marks) Start a New Writing Booklet. **Marks**

a) The fourth term of a geometric sequence is 96 and the seventh term is 12.

Find the

(i) first term and common ratio. 2

(ii) first term smaller than 0.0001 . 2

b) Find the equation of the tangents to the curve $y = 4\cos x$ at the point where $x = \dfrac{\pi}{6}$ 3

c) Annie was born on the 1st January 2000. Her parents invest $1000 on this day and on every birthday thereafter. The interest is paid at 6% compounded annually. After completing her HSC she decides to use the account to fund a gap year. She withdraws all the funds on 31/12/17 (getting paid her interest for 2017).

(i) What is the value of the investment on the 31/12/01 (after the interest for 2001 is paid)? 2

(ii) How much does Annie collect on 31/12/17? 3

d) Solve $2\log_2 x - \log_2(2x+6) = 1$. 3

e) Find:

(i) $\displaystyle\int 2\sin\left(\frac{\pi}{4} + x\right)dx$ 1

(ii) $\displaystyle\int \frac{x}{x^2+3}\ dx$ 2

Question 15 (18 marks) Start a New Writing Booklet. **Marks**

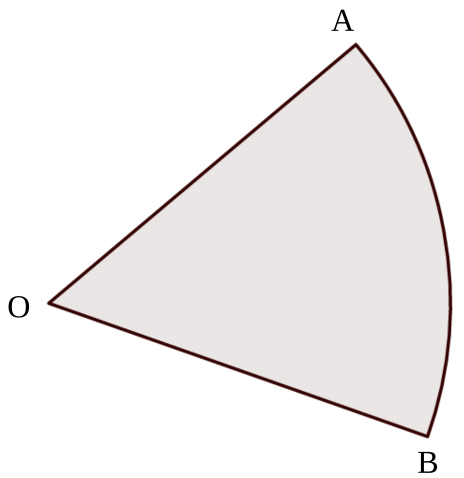
a) Evaluate $\lim\limits_{x \to 2} \dfrac{x^2-4}{x-2}$ 2

b) For what value of n is $\dfrac{6^{3n} \times 9^{n+1}}{8^n} = 1$. 2

c) The parabola $y = ax^2 + bx + c$ passes through the points $(0, 5)$, $(1, 3)$ and $(-1, 5)$. 3

Find the value of a, b and c .

d) 2

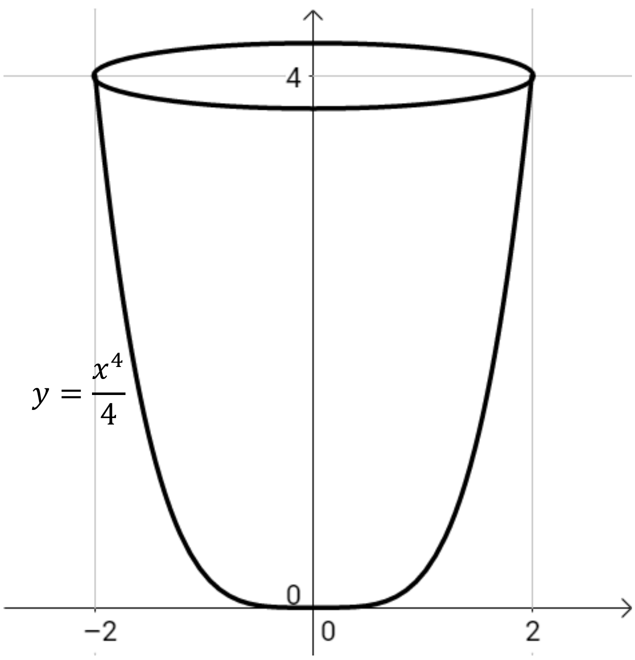


Arc AB is subtended by an angle of 72^0 at the centre of a circle radius 8cm.

(i) Calculate the length AB.

(ii) Calculate the area of sector AOB

e) 4



A bowl is formed by rotating the curve $y = \dfrac{x^4}{4}$ between x=0 and x=2 about the y axis. Find the volume of the bowl.

f) A cylindrical container closed at both ends is made from thin sheet metal. The container is to have a radius of r cm and height of h cm, such that its volume is 1000π cm^3.

[So $V = \pi r^2 h$ and $SA = 2\pi r^2 + 2\pi rh$]

(i) Show that the area of sheet metal required to make the container is $\left(2\pi r^2 + \frac{2000\pi}{r}\right)$ cm^2 1

(ii) Hence find the minimum area of sheet metal required to make the container. 4

End of Examination

Mathematics

Factorisation

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Angle sum of a polygon

$$S = (n-2) \times 180^\circ$$

Equation of a circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Trigonometric ratios and identities

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

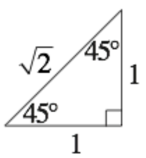
$$\sec\theta = \frac{1}{\cos\theta}$$

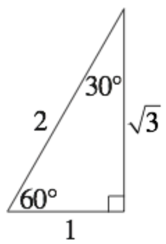
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Exact ratios





Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Perpendicular distance of a point from a line

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Slope (gradient) of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-gradient form of the equation of a line

$$y - y_1 = m(x - x_1)$$

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*n*th term of an arithmetic series

$$T_n = a + (n-1)d$$

Sum to *n* terms of an arithmetic series

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] \quad \text{or} \quad S_n = \frac{n}{2}(a+l)$$

*n*th term of a geometric series

$$T_n = ar^{n-1}$$

Sum to *n* terms of a geometric series

$$S_n = \frac{a(r^n - 1)}{r-1} \quad \text{or} \quad S_n = \frac{a(1 - r^n)}{1-r}$$

Limiting sum of a geometric series

$$S = \frac{a}{1-r}$$

Compound interest

$$A_n = P\left(1 + \frac{r}{100}\right)^n$$

Mathematics (continued)

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivatives

If $y = x^n$, then $\dfrac{dy}{dx} = nx^{n-1}$

If $y = uv$, then $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

If $y = \dfrac{u}{v}$, then $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

If $y = F(u)$, then $\dfrac{dy}{dx} = F'(u)\dfrac{du}{dx}$

If $y = e^{f(x)}$, then $\dfrac{dy}{dx} = f'(x)e^{f(x)}$

If $y = \log_e f(x) = \ln f(x)$, then $\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$

If $y = \sin f(x)$, then $\dfrac{dy}{dx} = f'(x)\cos f(x)$

If $y = \cos f(x)$, then $\dfrac{dy}{dx} = -f'(x)\sin f(x)$

If $y = \tan f(x)$, then $\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$

Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$\alpha + \beta = -\dfrac{b}{a}$ $\qquad$ $\alpha\beta = \dfrac{c}{a}$

Equation of a parabola

$$(x-h)^2 = \pm 4a(y-k)$$

Integrals

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int e^{ax+b} \, dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2}\left[f(a) + f(b)\right]$$

Simpson's rule (one application)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Logarithms – change of base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Angle measure

$180° = \pi$ radians

Length of an arc

$l = r\theta$

Area of a sector

$$\text{Area} = \frac{1}{2}r^2\theta$$

Student Name: ____________________________ Class Teacher: ________________________

Section I

Year 12 Trial HSC Examination 2017

Mathematics

Multiple-choice Answer Sheet - Questions 1 – 10

Allow about 15 minutes for this section.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ⊗ B ⊗ C ○ D ○

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

Student Name: ______________________ Class Teacher: ____________________

Section I

Year 12 Trial HSC Examination 2017

Mathematics

Multiple-choice Answer Sheet - Questions 1 – 10

Allow about 15 minutes for this section.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ~~●~~ B ~~●~~ C ○ D ○

1.	A ○	B ○	C ●	D ○
2.	A ○	B ○	C ●	D ○
3.	A ○	B ●	C ○	D ○
4.	A ○	B ●	C ○	D ○
5.	A ●	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ●
7.	A ○	B ○	C ●	D ○
8.	A ○	B ○	C ●	D ○
9.	A ○	B ●	C ○	D ○
10.	A ○	B ○	C ○	D ●

1) $y = x(x^2 - 1)$

$0 = x(x-1)(x+1)$

$\therefore x = 0, 1, -1 \qquad \therefore 3\text{pts}$ C

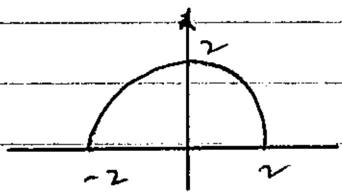
2) $a > 0 \qquad b^2 - 4ac < 0$

$7^2 - 4(1)(c) < 0$

$49 - 4c < 0$

$c > 12\frac{1}{4}$ C

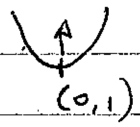
3) $y = \sqrt{4 - x^2}$



RANGE B

4) $x^2 = 4a(y - k)$

$4a = 8 \qquad \therefore a = 2$



$S(0,3)$ B

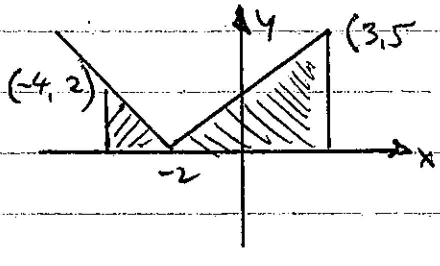
5) $y = \tan x \qquad y = \tan 6x$

D: $-\frac{\pi}{2} < x < \frac{\pi}{2} \qquad -\frac{\pi}{2} < 6x < \frac{\pi}{2}$

R: all real $y \qquad -\frac{\pi}{12} < x < \frac{\pi}{12}$

$\therefore \frac{\pi}{6}$ A

6) $\int_{-4}^{3} |x+2| \, dx$



$2 + 12\frac{2}{2}$

$= \frac{29}{2}$ D

7) $y = xe^{2x}$

$y' = 1 \cdot e^{2x} + 2x e^{2x}$

$= e^{2x}(1 + 2x)$ C

8) $|2x+4| = -x+4$

$2x+4 = -x+4$ or $2x+4 = -(-x+4)$

$3x = 0$ $\quad$ $2x+4 = x-4$

$x = 0$ $\quad$ $x = -8$

LHS = 4 $\quad$ RHS = 4 ✓ $\quad$ LHS = 12 $\quad$ RHS = 12 ✓

$\therefore$ 2 sol^n $\quad$ C

9) $f(x) = \dfrac{3x^4 - x}{x^2}$

$= 3x^2 - x^{-1}$

$f'(x) = 6x + x^{-2}$

$f'(1) = 6 + 1$

$= 7$ $\quad$ B

10) $\sum_{r=1}^{5} (-1)^r 2^r$

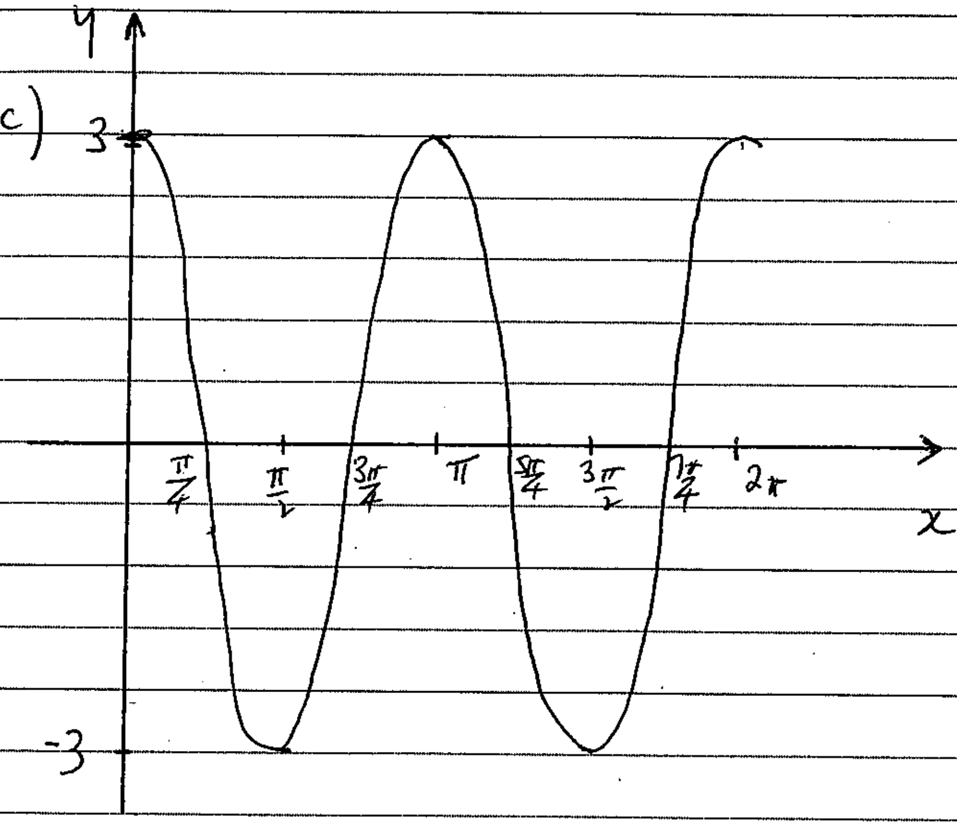
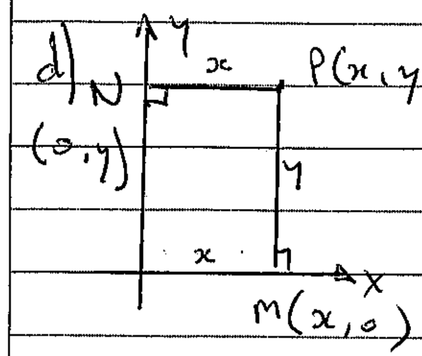
$= -1(2)^1 + (-1)^2(2)^2 + (-1)^3(2)^3 + (-1)^4(2)^4 + (-1)^5(2)^5$

$= -2 + 4 - 8 + 16 - 32$

$= 20 - 42$

$= -22$ $\quad$ D

MATHEMATICS– QUESTION 11 2017 HSC TRIAL . 2U

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $\sqrt{75} - \frac{1}{2}\sqrt{48} = 5\sqrt{3} - \frac{1}{2}(4\sqrt{3})$	$\frac{1}{2}$	
$= 3\sqrt{3}$	$\frac{1}{2}$	
b) $\sec 40^\circ 15' = \frac{1}{\cos 40^\circ 15'}$		
$= 1.31$	1	1.3 $\frac{1}{2}$ MARK.
c) 	2.	FULL MARKS FOR CORRECT PERIOD, AMPLITUDE, SHAPE $1\frac{1}{2}$ $0 \le \theta \le \pi$ $1\frac{1}{2}$ if 2 or less x intercepts shown NOTE SCALE
d)  $2PN = PM$ $4PN^2 = PM^2$ $4x^2 = y^2$ $y = \pm 2x$	1	or $\lvert y \rvert = \lvert 2x \rvert$ $y = \pm 2x$ or $y = \pm\lvert 2x \rvert$ [POORLY DONE] $\frac{1}{2}$ MARK for $y = 2x$ with some supporting working.
Locus is two straight lines with gradients ± 2 passing through $(0,0)$		

MATHEMATICS– QUESTION 11 (cont)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
e) $x^2+4x+y^2-6y-12=0$		
$x^2+4x+4+y^2-6y+9=12+4+9$	1	
$(x+2)^2+(y-3)^2=25$		Mostly well done
$\therefore$ Radius $=5$	1	
f) $\dfrac{x+3}{\frac{1}{x}+\frac{1}{3}}\times\dfrac{3x}{3x}=\dfrac{3x(x+3)}{3+x}$		Many did unusual things when trying to take reciprocals
$=3x$	1	
g) i) $y=\log_e\sqrt{3x^2-2}$		
Let $m=(3x^2-2)^{1/2}$		
$\dfrac{dm}{dx}=\frac{1}{2}(3x^2-2)^{-1/2}.6x$		
$=3x(3x^2-2)^{-1/2}$		
$y=\log_e m$		
$\dfrac{dy}{dm}=\dfrac{1}{m}$		
$\dfrac{dy}{dx}=\dfrac{dy}{dm}\times\dfrac{dm}{dx}$		
$=\dfrac{1}{m}\times 3x(3x^2-2)^{-1/2}$		
$=\dfrac{3x}{3x^2-2}$		

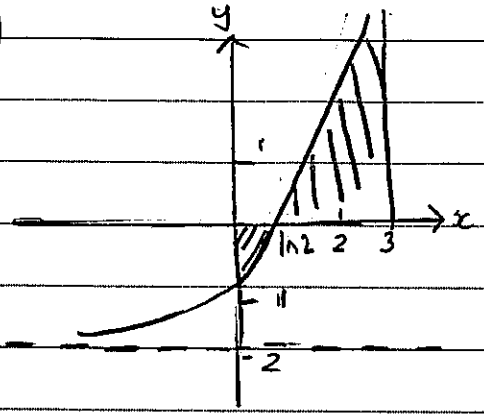
MATHEMATICS– QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
OR $y = \log_e \sqrt{3x^2-2}$		This is a better method.
$= \frac{1}{2}\log_e(3x^2-2)$		
$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{3x^2-2} \cdot 6x$		
$= \frac{3x}{3x^2-2}$	2	
ii) $y = \frac{x+3}{2x-5}$		
$u = x+3 \qquad v = 2x-5$		
$u' = 1 \qquad v' = 2$	1	
$\frac{dy}{dx} = \frac{1(2x-5) - 2(x+3)}{(2x-5)^2}$	1	
$= \frac{2x-5-2x-6}{(2x-5)^2}$		ERRORS FROM HERE ON IGNORED.
$= \frac{-11}{(2x-5)^2}$		
h) $LHS = \frac{1}{\sin\theta\cos\theta} - \tan\theta$		
$= \frac{1}{\sin\theta\cos\theta} - \frac{\sin\theta}{\cos\theta}$		
$= \frac{1}{\sin\theta\cos\theta} - \frac{\sin^2\theta}{\sin\theta\cos\theta}$		
$= \frac{1-\sin^2\theta}{\sin\theta\cos\theta}$	1	Well done by most
$= \frac{\cos^2\theta}{\sin\theta\cos\theta}$		

MATHEMATICS– QUESTION 11 cont

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$= \dfrac{\cos\theta}{\sin\theta}$		
$= \cot\theta$	1	
$=$ RHS.		
i) i) $y = 2x^3 - 12x^2 + 18x + 3$		
$y' = 6x^2 - 24x + 18$		
$= 6(x^2 - 4x + 3)$		MUST HAVE 6.
$= 6(x-3)(x-1)$	1	
$y' = 0$ when $x = 1$ or 3		
$y'' = 12x - 24$		
when $x = 1$ $y'' = -12$		
< 0 $\cap$		
$\therefore$ max t.p $(1,5)$	1	
when $x = 3$ $y'' = 12$		
> 0 $\cup$		
$\therefore$ min tp $(3,-3)$	1	
ii) Max value at end points of domain or at local maxima		
$x = 5$ $y = 2(5)^3 - 12(5)^2 + 18(5) - 3$		Many students
$= 37$		did not seem to
$x = -5$ $y = 2(-5)^3 - 12(-5)^2 + 18(-5) - 3$		know max/min needs
< 0		to be checked at
$\therefore$ Max value is 37	1	end of domain

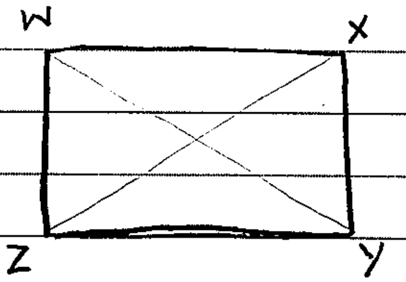
MATHEMATICS– QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
12 a) $x > 0$	1	
b)  when $y=0$ $e^x = 2$ $x = \ln 2$ $\doteqdot 0.69\ldots$		This question was poorly done. Stress to students to ALWAYS draw a diagram.
$A = \left\| \int_0^{\ln 2} (e^x - 2)\,dx \right\| + \int_{\ln 2}^{3} (e^x - 2)\,dx$	1	
$= \left\| \left[e^x - 2x \right]_0^{\ln 2} \right\| + \left[e^x - 2x \right]_{\ln 2}^{3}$		Took off ½ mark for silly errors
$= \left\| e^{\ln 2} - 2\ln 2 - (e^0 - 2(0)) \right\| + \left[e^3 - 6 - (e^{\ln 2} - 2\ln 2) \right]$	1	
$= \left\| 2 - 2\ln 2 - 1 \right\| + e^3 - 6 - 2 + 2\ln 2$		Students received ½/3 if they had the answer $e^3 - 7$ or $13.085\ldots$ and did not take into account the negtive area.
$= \left\| 1 - 2\ln 2 \right\| + e^3 - 8 + 2\ln 2$ since $1 - 2\ln 2 < 0$		
$= 2\ln 2 - 1 + e^3 - 8 + 2\ln 2$	1	
$= 4\ln 2 + e^3 - 9$ units^2		
accepted $13.86\ u^2$ (to 2dp)		

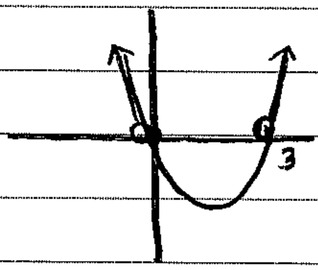
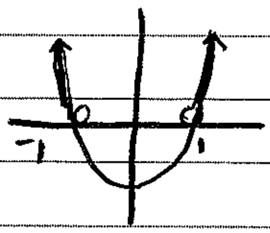
MATHEMATICS– QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) (i) $T_n = a + (n-1)d$		
$= 400 + (n-1)(-50)$	1	
$= 450 - 50n$		
or $5(9-n)$		
(ii) $T_n < 0$		
$450 - 50n < 0$		
$450 < 50n$		
$9 < n$		
$n > 9$	1	
$\therefore n = 10$		
The 10^{th} term is the first negative term	1	
$T_{10} = 400 + (10-1)(-50)$		
$= -50$		
(iii) $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$		
$= \frac{20}{2}\left[2(400) + (20-1)(-50)\right]$		
$= -1500$	1	
d) $a = t - 2$		
$v = \frac{t^2}{2} - 2t + c$ $t = 0,\ v = 3$	½	
$3 = \frac{0^2}{2} - 2(0) + c$		
$c = 3$		
$\therefore v = \frac{t^2}{2} - 2t + 3$	½	

MATHEMATICS– QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$x = \frac{t^3}{6} - t^2 + 3t + c \qquad t=0,\ x=1$	½	
$1 = \frac{0^3}{6} - 0^2 + 3(0) + c$		
$c = 1$		
$\therefore x = \frac{t^3}{6} - t^2 + 3t + 1$	½	
When $t=6$		
$x = \frac{6^3}{6} - 6^2 - 3(6) + 1$		
$x = 19$	1	
$\therefore$ Position of the particle is 19m to the right of the origin.		
e) 		Poorly done.
In ΔZWY and ΔZXY		A lot of students chose incorrect triangles to prove congruency. They still achieved marks for knowing congruency proofs and correct final statement.
$WZ = XY$ (opposite sides rectangle equal)	½	
ZY is common	½	
$\angle WZY = \angle XYZ = 90^\circ$ (all angles 90° in a rectangle)	½	
$\Delta ZWY \equiv \Delta ZXY$ (SAS)	½	
$\therefore XZ = WY$ (corresponding sides in congruent triangles)	1	

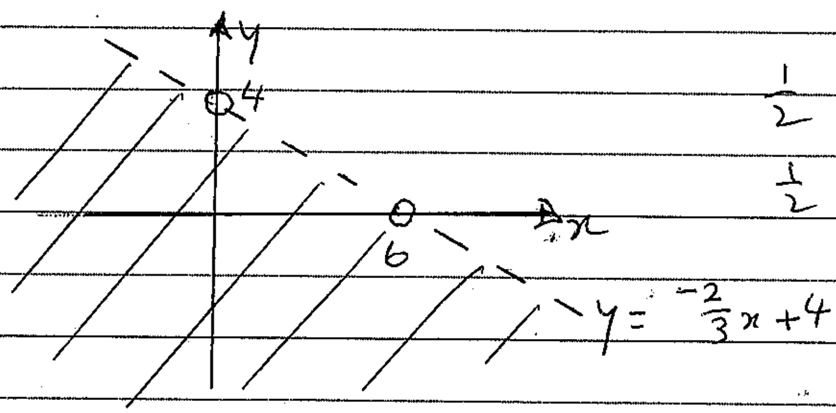
MATHEMATICS – QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
OR		
$\angle ZWX = \angle XYZ = 90°$ (all angles $90°$ in a rectangle)	1	
In ΔWXZ		
$XZ^2 = WZ^2 + WX^2$ (By Pythagoras)	1	
In ΔWYZ		
$WY^2 = WZ^2 + ZY^2$		
Now $WX = ZY$ (opposite sides rectangle equal)	1	
$\therefore WY^2 = WZ^2 + WX^2$		
$WY^2 = XZ^2$		
$\therefore WY = XZ$		
f) $x^2 - 3x > 0$		If $x^2 > 3x$
$x(x-3) > 0$		$x > 3$
considering both solutions. 1 		only 1/2
$\therefore x > 3$ or $x < 0$		
½ ½		
g) $y' = 3x^2 - 3$ increasing $y' > 0$		
$3(x^2-1) > 0$		Only 1/2
$3(x-1)(x+1) > 0$	1	if equality sign used.
 $\therefore x > 1$ or $x < -1$	1	

MATHEMATICS– QUESTION 13 2017 2U TRIAL

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) i) $2x + 3y = 12$		Generally well done
$y = 0 \quad x = 6 \quad B(6, 0)$		
$x = 0 \quad y = 4 \quad A(4, 0)$	$\frac{1}{2}$	
$AB^2 = 6^2 + 4^2$		A few students wrote distance formula incorrectly
$= 52$		
$AB = \sqrt{52}$		
$= 2\sqrt{13}$	$\frac{1}{2}$	Surd should be simplified.
ii) $m_{AB} = \frac{0-4}{6-0}$	$\frac{1}{2}$	Use $\frac{\text{rise}}{\text{run}}$ or $\frac{y_2-y_1}{x_2-x_1}$ not other methods.
$= -\frac{2}{3}$		
$m_{DC} = \frac{3}{2}$	$\frac{1}{2}$	
$\frac{3}{2} = \frac{p-2}{4-1}$	$\frac{1}{2}$	
$9 = 2p - 4$		Some students did not use $AB \perp DC$ then $m_{AB} \times m_{DC} = -1$.
$13 = 2p$		
$p = 6\frac{1}{2}$	$\frac{1}{2}$	
OR $y - 2 = \frac{3}{2}(x - 1)$	$\frac{1}{2}$ $m = \frac{3}{2}$	Usually well done.
$2y - 4 = 3x - 3$		
$0 = 3x - 2y + 1$		
$C(4, p)$		
$0 = 12 - 2p + 1$	1	
$2p = 13$		
$p = 6\frac{1}{2}$	$\frac{1}{2}$	

MATHEMATICS– QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
iii) $d = \dfrac{\lvert 2(1)+3(2)-12 \rvert}{\sqrt{4+9}}$	1	Some students did not know how to apply this formula.
$= \dfrac{\lvert 2+6-12 \rvert}{\sqrt{13}}$		
$= \dfrac{4\sqrt{13}}{13}$	1	or $\dfrac{4}{\sqrt{13}}$
iv) $A = \frac{1}{2} \times 2\sqrt{13} \times \frac{4}{\sqrt{13}}$	$\frac{1}{2}$	
$= 4$ Area is 4 units^2	$\frac{1}{2}$	
v) 	$\frac{1}{2}$ mark dotted line $\frac{1}{2}$ shading.	
b) $0 \le \theta \le 2\pi$ $0 \le 2\theta \le 4\pi$	$\frac{1}{2}$ mark any correct answer	Not well done. Majority only had 2 answers
$\sin 2\theta = -\frac{\sqrt{3}}{2}$ Quadrants 3, 4, 7, 8 First quadrant related angle $\frac{\pi}{3}$	1 all 1 4	4 values 2θ correct answers.
$2\theta = \pi + \frac{\pi}{3},\ 2\pi - \frac{\pi}{3},\ 3\pi + \frac{\pi}{3},\ 4\pi - \frac{\pi}{3}$		
$= \frac{4\pi}{3},\ \frac{5\pi}{3},\ \frac{10\pi}{3},\ \frac{11\pi}{3}$		

$\theta = \frac{2\pi}{3},\ \frac{5\pi}{6},\ \frac{5\pi}{3},\ \frac{11\pi}{6}$

MATHEMATICS EXTENSION I – QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) i) $\alpha+\beta = 4/3$	1	Some errors with signs.
ii) $\alpha\beta = \frac{-1}{3}$	1	
iii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$	$\frac{1}{2}$	
$= \frac{16}{9} + \frac{2}{3}$		
$= \frac{22}{9}$	$\frac{1}{2}$	
d) $A \approx \frac{4}{6}(\ln 1 + 4\ln 3 + \ln 5) + \frac{4}{6}(\ln 5 + 4\ln 7 + \ln 9)$	1	Some did not know $\log_e x = \ln x$
$= \frac{2}{3}(0 + 4\ln 3 + \ln 5) + \frac{2}{3}(\ln 5 + 4\ln 7 + \ln 9)$		Some mixed up formula.
≈ 11.73	1	
$= 12$	1 (sig fig)	
e) $x = 6t - t^3$		1 for differentiation and $\dot{x}=0$
$\dot{x} = 6 - 3t^2$		
$\dot{x}=0$ $\quad 3t^2 = 6$	1	
$t = \pm\sqrt{2}$ as $t \geq 0$	$\frac{1}{2}$	
$t = \sqrt{2}$	$\frac{1}{2}$	
$\ddot{x} = -6t$		
at $t=\sqrt{2}$ $\quad \ddot{x} = -6\sqrt{2}$		
Acceleration $-6\sqrt{2}$ cm/s^2	1	

MATHEMATICS– QUESTION 14		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

(a)(i) GP: $T_4 = ar^3 = 96$

$T_7 = ar^6 = 12$

$\frac{T_7}{T_4} = \frac{ar^6}{ar^3} = \frac{96}{12}$

$r^3 = \frac{1}{8}$

$\therefore r = \frac{1}{2}$ — 1 for correct common ratio

$a\left(\frac{1}{8}\right) = 96$

$a = 96 \times 8$

$\therefore a = 768$ — 1 for correct first term

(ii) $ar^{n-1} < 0.0001$

$768\left(\frac{1}{2}\right)^{n-1} < 0.0001$

$\left(\frac{1}{2}\right)^{n-1} < \frac{0.0001}{768}$ — 1

$n-1\left(\ln\frac{1}{2}\right) < \ln\left(\frac{0.0001}{768}\right)$

$n-1 > \frac{\ln\left(\frac{0.0001}{768}\right)}{\ln\left(\frac{1}{2}\right)}$

NOTE: inequality sign reverses as $\ln\frac{1}{2} < 0$

$n > \frac{\ln\left(\frac{0.0001}{768}\right)}{\ln\left(\frac{1}{2}\right)} + 1$

$n > 23.87267488$ — $\frac{1}{2}$

$n = 24$

$\therefore T_{24}$ is the first term < 0.0001 — $\frac{1}{2}$ for correct answer.

($1\frac{1}{2}$) marks if the inequality sign was not reversed, but everything else correct.

MATHEMATICS– QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(b) $y = 4\cos x$ at $x = \frac{\pi}{6}$ $(30°)$		
$y = 4\cos\frac{\pi}{6}$		
$\frac{dy}{dx} = -4\sin x$ $\quad = \frac{4\sqrt{3}}{2}$		
at $x = \frac{\pi}{6}$, $\quad y = 2\sqrt{3}$ — 1		for correct y coordinate
$\frac{dy}{dx} = -4\sin\frac{\pi}{6}$		
$= -4 \times \frac{1}{2}$		
$= -2$ — 1 for correct gradient		
$\therefore$ Equation of tangent:		
$y - 2\sqrt{3} = -2(x - \frac{\pi}{6})$		
$y - 2\sqrt{3} = -2x + \frac{\pi}{3}$		
$y = -2x + \frac{\pi}{3} + 2\sqrt{3}$ — 1 for correct equation		
(c)(i) let A_n be the value of the investment after n years.		
So A_1 is at 31/12/2000		
A_2 is at 31/12/2001		
$A_1 = 1000(1.06)$		
$= \$1060$ — 1		
$A_2 = A_1 + A_1(1.06)$		
$= 1000(1.06) + 1000(1.06)^2$		
$= 1000(1.06)(1 + 1.06)$		
$= \$2183.60$ — 1		
$\therefore$ Value on 31/12/2001 is \$2183.60		

MATHEMATICS– QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(ii) $A_1 \rightarrow 31/12/2000$		
$A_2 \rightarrow 31/12/2001$		
$\vdots$		
$A_{18} \rightarrow 31/12/2017$		
$\therefore A_{18} = 1000(1.06)^1 + 1000(1.06)^2 + \cdots + 1000(1.06)^{18}$ — (1)		
$= 1000(1.06)\left[1 + 1.06 + 1.06^2 + \cdots + 1.06^{17}\right]$		→ a GP:
$= 1000(1.06)\left[\dfrac{1(1.06^{18} - 1)}{1.06 - 1}\right]$ — (1)		$a = 1$ $r = 1.06$ $n = 18$
$= 32\,759.9917$		
$= \$32\,760.00$ — (1)		
($2\frac{1}{2}$ marks for A_{17} with correct working).		
(d) $2\log_2 x - \log_2(2x+6) = 1$		
$\log_2\left(\dfrac{x^2}{2x+6}\right) = \log_2 2$ — (1)		for applying correct log laws.
$\therefore \dfrac{x^2}{2x+6} = 2$		
$x^2 = 4x + 12$		[$2\frac{1}{2}$ marks for $x=-2$ and $x=6$ with no other conclusion]
$x^2 - 4x - 12 = 0$ — (1)		
$(x+2)(x-6) = 0$		
$x = -2$ OR $x = 6$		
Test $x = -2$: $2\log(-2)$ is not defined		
$\therefore x = -2$ is not a solution		
Test $x = 6$: $2\log_2 6 - \log_2 18 = \log_2 \dfrac{6^2}{18} = \log_2 2 = 1$		
$\therefore x = 6$ is the only solution. — (1)		

MATHEMATICS– QUESTION		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(e) (i) $\int 2\sin\left(\frac{\pi}{4}+x\right)dx$		
$= 2 \times \ -\cos\left(\frac{\pi}{4}+x\right)+C$		
$= -2\cos\left(\frac{\pi}{4}+x\right)+C$ — ① for correct answer.		
(ii) $\frac{1}{2}\int \frac{2x}{x^2+3}\,dx$ — ① for showing $\frac{1}{2}\times 2$		
$= \frac{1}{2}\ln(x^2+3)+C$ — ① for correct answer		
① mark for $\ln(x^2+3)+C$		
① mark for $2\ln(x^2+3)+C$		

MATHEMATICS – QUESTION 15 (18 marks)		
SUGGESTED SOLUTIONS	**MARKS**	**MARKER'S COMMENTS**
pg1.		
(a) $\lim_{x \to 2} \dfrac{x^2-4}{x-2}$	2	generally well done some students did not set this out correctly please improve setting out in future
$= \lim_{x \to 2} \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$ (1)		
$= \lim_{x \to 2} x+2$		
$= 2+2$		
$= 4.$ (1)		
(b) $\dfrac{6^{3n} \times 9^{n+1}}{8^n} = 1$	2	there was a few different strategies used. Very few students using logarithms solve the equation successfully. Index laws were not correctly used. This topic would benefit from revision.
$\dfrac{(2 \times 3)^{3n} \times (3^2)^{n+1}}{(2^3)^n} = 1$ (1)		
$\dfrac{2^{3n} \cdot 3^{3n} \cdot 3^{2n+2}}{2^{3n}} = 1$		
$\dfrac{2^{3n} \cdot 3^{5n+2}}{2^{3n}} = 1$		
$3^{5n+2} = 1$		
$3^{5n+2} = 3^0$		
$5n+2 = 0$		
$n = \dfrac{-2}{3}.$ (1)		

MATHEMATICS – QUESTION 15		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
pg2.		
c) $y = ax^2 + bx + c$ —— ① $(0,5)$ $(1,3)$ $(-1,5)$	3	generally well done
substituting $(0,5)$ in ①		
$5 = a(0)^2 + b(0) + c$		
$\boxed{5 = c}$ (1)		
$\therefore y = ax^2 + bx + 5$ —— ②		
substituting, $\left.\begin{matrix} x = 1 \\ y = 3 \end{matrix}\right\}$ in ②		
$3 = a + b + 5$		
$a + b = -2$ —— ③		
substituting, $\left.\begin{matrix} x = -1 \\ y = 5 \end{matrix}\right\}$ in ②		
$5 = a(-1)^2 + b(-1) + 5$		
$5 = a - b + 5$		
$a - b = 0$ —— ④		
③ + ④ $a + b = -2$		
$a - b = 0$		
$2a = -2$		
$\boxed{a = -1}$ (1)		
sub $a = -1$ in ④		
$a - b = 0$		
$a = b$		
$\therefore -1 = b$		
$\boxed{\therefore b = -1}$ (1)		
$\therefore a = -1, b = -1, c = 5$		
and the equation is		
$y = -x^2 - x + 5.$		

MATHEMATICS – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

(d) pg3

angles in radian measure.

$360^\circ = 2\pi^c$

$1^\circ = \frac{2\pi}{360}^c$

$72^\circ = \frac{2\pi}{360} \times 72^c$

$= \frac{144\pi}{360}$

$= \frac{2\pi}{5}$

(i) $l_{arc} = r\theta$ — (1)

$= 8 \times \frac{2\pi}{5}$

$= \frac{16\pi}{5}$ cm.

(ii) $A_{sector} = \frac{1}{2} r^2 \theta$ — (1)

$= \frac{1}{2} \times 8^2 \times \frac{2\pi}{5}$

$= \frac{64\pi}{5}$ cm^2.

Method 2 (as in junior school)

$l_{arc} = \frac{\theta}{360} \times 2\pi r$

$= \frac{72}{360} \times 2 \times \pi \times 8$

$= \frac{16\pi}{5}$ cm

$Area_{sector} = \frac{\theta}{360} \times \pi r^2$

$= \frac{72}{360} \times \pi \times 8^2$

$= \frac{64\pi}{5}$ cm^2.

Marker's comments:

- It was very well done by most students. However, a significant number of students, incorrectly tried to use degrees in the formulas $l = r\theta$ and $A = \frac{1}{2}r^2\theta$.

MATHEMATICS – QUESTION 15 (18 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
pg 4		
(e) $y = \frac{x^4}{4}$ $y = \frac{x^4}{4}$	4.	
when $x=0$, $y=0$		
$4y = x^4$ $x = \cancel{2}\,y = \frac{\cancel{2}4}{4}$		
$\therefore x^2 = \sqrt{4y}$ $= 4$		
$x^2 = 2\sqrt{y}$ $\therefore$		
$x = 2y^{\frac{1}{2}}$		
$V = \pi \int_0^4 y^2 \, dx$ (1)		
$= \pi \int_0^4 2y^{\frac{1}{2}} \, dx$ (1)		
$= 2\pi \int_0^4 y^{\frac{1}{2}} \, dx.$		
$= 2\pi \left[\frac{2}{3} y^{\frac{3}{2}}\right]_0^4$ integrating correctly (1)		
$= \frac{4\pi}{3}\left[y^{\frac{3}{2}}\right]_0^4$		
$= \frac{4\pi}{3}\left[4^{\frac{3}{2}} - 0^{\frac{3}{2}}\right]$		
$= \frac{4\pi}{3}\left[2^3 - 0\right]$		
$= \frac{32\pi}{3}$ evaluating integral. (1)		
(f) $V = \pi r^2 h$ $S.A = 2\pi r^2 + 2\pi r h$	(1)	Show all
$\frac{1000\,\cancel{\pi}}{\cancel{\pi} r^2} = \frac{\pi r^2 h}{\pi r^2}$ $= 2\pi r^2 + 2\pi r \cdot \frac{1000}{r^2}$		necessary steps
$= 2\pi r^2 + \frac{2000\pi}{r}$		to receive full
$\therefore h = \frac{1000}{r^2}$		marks.

MATHEMATICS – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
pg5.		
f (ii). $A = 2\pi r^2 + \frac{2000\pi}{r}$	4	generally well done.
$= 2\pi r^2 + 2000\pi r^{-1}$		Most problems for students occurred
$\frac{dA}{dr} = 4\pi r - 2000\pi r^{-2}$ (1)		when they were working with indices.
if $\frac{dA}{dr} = 0$,		setting out
$4\pi r - 2000\pi r^{-2} = 0$		could also be
$4\pi r = \frac{2000\pi}{r^2}$		better. Ask for another
$\frac{4\pi r^3}{4\pi} = \frac{2000\pi}{4\pi}$		booklet, even if it is your
$r^3 = 500$		last question,
$r = \sqrt[3]{500}$ (1)		rather than squashing
$\frac{d^2A}{dr^2} = 4\pi + 4000\pi r^{-3}$		you work and perhaps
when $r = \sqrt[3]{500}$		causing mistakes.
$\frac{d^2A}{dr^2} = 4\pi \times 4000\pi\left[(500)^{\frac{1}{3}}\right]^{-3}$		
$= 4\pi + 4000\pi\left[500^{\frac{1}{3}\times -3}\right]$		
$= 4\pi + 4000\pi\left[500^{-1}\right]$		
$= 4\pi + \frac{400\cancel{0}\pi}{50\cancel{0}}$		
$= 12\pi$ which is positive (1)		
$\therefore$ when $n = \sqrt[3]{500}$ there is a minimum		
where $r = \sqrt[3]{500}$ $A = 2\pi r^2 + \frac{2000\pi}{r}$		
$= 200(500)^{\frac{2}{3}} + 2000\pi\,(5000)^{-\frac{1}{3}}$		
$= \frac{3000\pi}{\sqrt[3]{500}}$ cm^3 $= 1187$ cm^3 (nearest cm3)		